Calcul des déformations des barres élastiques

Barres élastiques en arc de cercle - Forces et couples concentrés

Torseur des forces de cohésion

$$\mathbf{r_F}(\psi_F) := \begin{pmatrix} R\cdot\cos(\psi_F) \\ R\cdot\sin(\psi_F) \\ 0 \end{pmatrix} \qquad \mathbf{r_S}(\alpha') := \begin{pmatrix} R\cdot\cos(\alpha') \\ R\cdot\sin(\alpha') \\ 0 \end{pmatrix} \qquad \mathbf{r_v}(\alpha) := \begin{pmatrix} R\cdot\cos(\alpha) \\ R\cdot\sin(\alpha) \\ 0 \end{pmatrix}$$

Forces et couples concentrés en ψ_F

$$\mathbf{M_c}(\psi_F, \alpha') := \left[\mathbf{C} + (\mathbf{r_F}(\psi_F) - \mathbf{r_S}(\alpha')) \times \mathbf{F}\right]\cdot(\alpha' \le \psi_F)$$

$$\mathbf{M_c}(\psi_F, \alpha') := \left[\begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} + R\cdot\begin{bmatrix} F_z\cdot(\sin(\psi_F) - \sin(\alpha')) \\ -F_z\cdot(\cos(\psi_F) - \cos(\alpha')) \\ -F_x\cdot(\sin(\psi_F) - \sin(\alpha')) + F_y\cdot(\cos(\psi_F) - \cos(\alpha')) \end{bmatrix}\right]\cdot(\alpha' \le \psi_F)$$

Sollicitations

$$\mathbf{e'_1}(\alpha') := (-\sin(\alpha') \quad \cos(\alpha') \quad 0)^T \qquad \mathbf{e'_2}(\alpha') := (-\cos(\alpha') \quad -\sin(\alpha') \quad 0)^T \qquad \mathbf{e'_3}(\alpha') := (0 \quad 0 \quad 1)^T$$

Moment de torsion $\qquad M_t(\psi_F, \alpha') := \mathbf{M_c}(\psi_F, \alpha')\cdot\mathbf{e'_1}(\alpha')$

Moments de flexion $\qquad M_{f2}(\psi_F, \alpha') := \mathbf{M_c}(\psi_F, \alpha')\cdot\mathbf{e'_2}(\alpha') \qquad M_{f3}(\psi_F, \alpha') := \mathbf{M_c}(\psi_F, \alpha')\cdot\mathbf{e'_3}(\alpha')$

Contraintes

k = limite d'élasticité en traction / limite d'élasticité en compression

M point de la section droite le plus éloigné de O sur l'axe O$\mathbf{e'_2}$

N point de la section droite le plus éloigné de O sur l'axe O$\mathbf{e'_3}$

$$\tau_M(\psi_F, \alpha') := \frac{M_t(\psi_F, \alpha')}{W_t} \qquad \tau_N(\psi_F, \alpha') := \frac{M_t(\psi_F, \alpha')}{W'_t}$$

$$\sigma_M(\psi_F, \alpha') := \frac{M_{f3}(\psi_F, \alpha')}{W_{f3}} \qquad \sigma_N(\psi_F, \alpha') := \frac{M_{f2}(\psi_F, \alpha')}{W_{f2}}$$

$$\sigma_{équiv\_M}(k, \psi_F, \alpha') := \frac{1-k}{2}\cdot\left|\sigma_M(\psi_F, \alpha')\right| + \frac{1+k}{2}\cdot\sqrt{\sigma_M(\psi_F, \alpha')^2 + 4\cdot\tau_M(\psi_F, \alpha')^2}$$

$$\sigma_{équiv\_N}(k, \psi_F, \alpha') := \frac{1-k}{2}\cdot\left|\sigma_N(\psi_F, \alpha')\right| + \frac{1+k}{2}\cdot\sqrt{\sigma_N(\psi_F, \alpha')^2 + 4\cdot\tau_N(\psi_F, \alpha')^2}$$

Calcul des déplacements par les intégrales de Mohr

Calcul des déplacements linéiques

Force unitaire virtuelle $\mathbf{v}(\lambda,\gamma) := (\cos(\lambda)\cdot\sin(\gamma) \quad \sin(\lambda)\cdot\sin(\gamma) \quad \cos(\gamma))^T$

Sollicitations dues à la force unitaire $\mathbf{M_v}(\alpha,\lambda,\gamma,\alpha') := \left[\left(\mathbf{r_v}(\alpha) - \mathbf{r_S}(\alpha')\right)\times \mathbf{v}(\lambda,\gamma)\right]\cdot(\alpha' < \alpha)$

$$\mathbf{M_v}(\alpha,\lambda,\gamma,\alpha') := R\cdot\begin{bmatrix} \cos(\gamma)\cdot(\sin(\alpha)-\sin(\alpha')) \\ -\cos(\gamma)\cdot(\cos(\alpha)-\cos(\alpha')) \\ -(\cos(\lambda)\cdot\sin(\gamma))\cdot(\sin(\alpha)-\sin(\alpha')) + \sin(\lambda)\cdot\sin(\gamma)\cdot(\cos(\alpha)-\cos(\alpha')) \end{bmatrix}\cdot(\alpha' < \alpha)$$

$$M_{tv}(\alpha,\lambda,\gamma,\alpha') := \mathbf{M_v}(\alpha,\lambda,\gamma,\alpha')\cdot\mathbf{e'_1}(\alpha') \qquad lim(\alpha,\psi_F) := \alpha\cdot(\alpha \le \psi_F) + \psi_F\cdot(\alpha > \psi_F)$$

$$M_{fv2}(\alpha,\lambda,\gamma,\alpha') := \mathbf{M_v}(\alpha,\lambda,\gamma,\alpha')\cdot\mathbf{e'_2}(\alpha') \qquad M_{fv3}(\alpha,\lambda,\gamma,\alpha') := \mathbf{M_v}(\alpha,\lambda,\gamma,\alpha')\cdot\mathbf{e'_3}(\alpha')$$

Déplacement dans la direction de **v** $\delta_{tv}(\psi_F,\alpha,\lambda,\gamma) := \dfrac{R}{G\cdot J_t}\cdot\displaystyle\int_0^{lim(\alpha,\psi_F)} M_t(\psi_F,\alpha')\cdot M_{tv}(\alpha,\lambda,\gamma,\alpha')\,d\alpha'$

$$\delta_{fv2}(\psi_F,\alpha,\lambda,\gamma) := \frac{R}{E\cdot I_{22}}\cdot\int_0^{lim(\alpha,\psi_F)} M_{f2}(\psi_F,\alpha')\cdot M_{fv2}(\alpha,\lambda,\gamma,\alpha')\,d\alpha'$$

$$\delta_{fv3}(\psi_F,\alpha,\lambda,\gamma) := \frac{R}{E\cdot I_{33}}\cdot\int_0^{lim(\alpha,\psi_F)} M_{f3}(\psi_F,\alpha')\cdot M_{fv3}(\alpha,\lambda,\gamma,\alpha')\,d\alpha'$$

$$\delta_v(\psi_F,\alpha,\lambda,\gamma) := \delta_{tv}(\psi_F,\alpha,\lambda,\gamma) + \delta_{fv2}(\psi_F,\alpha,\lambda,\gamma) + \delta_{fv3}(\psi_F,\alpha,\lambda,\gamma)$$

Calcul des déplacements angulaires

Couple unitaire virtuel $\mathbf{cv}(\lambda_c,\gamma_c) := (\cos(\lambda_c)\cdot\sin(\gamma_c) \quad \sin(\lambda_c)\cdot\sin(\gamma_c) \quad \cos(\gamma_c))^T$

Sollicitations dues au couple unitaire $\mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{cv}(\lambda_c,\gamma_c)\cdot(\alpha' < \alpha)$

$$\mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha') := \begin{pmatrix} \cos(\lambda_c)\cdot\sin(\gamma_c) \\ \sin(\lambda_c)\cdot\sin(\gamma_c) \\ \cos(\gamma_c) \end{pmatrix}\cdot(\alpha' < \alpha)$$

$$M_{tcv}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\mathbf{e'_1}(\alpha')$$

$$M_{fcv2}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\mathbf{e'_2}(\alpha') \qquad M_{fcv3}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\mathbf{e'_3}(\alpha')$$

Déplacement angulaire autour de l'axe défini par **v** $\theta_{tcv}(\psi_F,\alpha,\lambda_c,\gamma_c) := \dfrac{R}{G\cdot J_t}\cdot\displaystyle\int_0^{lim(\alpha,\psi_F)} M_t(\psi_F,\alpha')\cdot M_{tcv}(\alpha,\lambda_c,\gamma_c,\alpha')\,d\alpha'$

$$\theta_{fcv2}(\psi_F,\alpha,\lambda_c,\gamma_c) := \frac{R}{E\cdot I_{22}}\cdot\int_0^{lim(\alpha,\psi_F)} M_{f2}(\psi_F,\alpha')\cdot M_{fcv2}(\alpha,\lambda_c,\gamma_c,\alpha')\,d\alpha'$$

$$\theta_{fcv3}(\psi_F,\alpha,\lambda_c,\gamma_c) := \frac{R}{E\cdot I_{33}}\cdot\int_0^{lim(\alpha,\psi_F)} M_{f3}(\psi_F,\alpha')\cdot M_{fcv3}(\alpha,\lambda_c,\gamma_c,\alpha')\,d\alpha'$$

$$\theta_{cv}(\psi_F,\alpha,\lambda_c,\gamma_c) := \theta_{tcv}(\psi_F,\alpha,\lambda_c,\gamma_c) + \theta_{fcv2}(\psi_F,\alpha,\lambda_c,\gamma_c) + \theta_{fcv3}(\psi_F,\alpha,\lambda_c,\gamma_c)$$